FUNDAMENTAL THEOREM OF PROJECTIVE GEOMETRY 1.4.1. pdf

1: Fundamental theorem - Wikipedia

The fundamental theorem. The group PÎ“Ln(K) clearly acts on Tn(K). The following theorem will be proved in Â§2. Theorem 1 (Fundamental theorem of projective geometry). If K is a ï¬•eld.

Overview[edit] The Fundamental Theory of Projective Geometry Projective geometry is an elementary non-metrical form of geometry, meaning that it is not based on a concept of distance. In two dimensions it begins with the study of configurations of points and lines. That there is indeed some geometric interest in this sparse setting was first established by Desargues and others in their exploration of the principles of perspective art. The simplest illustration of duality is in the projective plane, where the statements "two distinct points determine a unique line" i. Projective geometry can also be seen as a geometry of constructions with a straight-edge alone. For example, the different conic sections are all equivalent in complex projective geometry, and some theorems about circles can be considered as special cases of these general theorems. During the early 19th century the work of Jean-Victor Poncelet , Lazare Carnot and others established projective geometry as an independent field of mathematics. After much work on the very large number of theorems in the subject, therefore, the basics of projective geometry became understood. The incidence structure and the cross-ratio are fundamental invariants under projective transformations. Projective geometry can be modeled by the affine plane or affine space plus a line hyperplane "at infinity" and then treating that line or hyperplane as "ordinary". Growth measure and the polar vortices. Based on the work of Lawrence Edwards In a foundational sense, projective geometry and ordered geometry are elementary since they involve a minimum of axioms and either can be used as the foundation for affine and Euclidean geometry. Mathematics and art The first geometrical properties of a projective nature were discovered during the 3rd century by Pappus of Alexandria. He made Euclidean geometry , where parallel lines are truly parallel, into a special case of an all-encompassing geometric system. The works of Gaspard Monge at the end of 18th and beginning of 19th century were important for the subsequent development of projective geometry. The work of Desargues was ignored until Michel Chasles chanced upon a handwritten copy during Meanwhile, Jean-Victor Poncelet had published the foundational treatise on projective geometry during Poncelet separated the projective properties of objects in individual class and establishing a relationship between metric and projective properties. The non-Euclidean geometries discovered soon thereafter were eventually demonstrated to have models, such as the Klein model of hyperbolic space , relating to projective geometry. This early 19th century projective geometry was intermediate from analytic geometry to algebraic geometry. When treated in terms of homogeneous coordinates , projective geometry seems like an extension or technical improvement of the use of coordinates to reduce geometric problems to algebra , an extension reducing the number of special cases. The work of Poncelet , Jakob Steiner and others was not intended to extend analytic geometry. Techniques were supposed to be synthetic: As a result, reformulating early work in projective geometry so that it satisfies current standards of rigor can be somewhat difficult. Even in the case of the projective plane alone, the axiomatic approach can result in models not describable via linear algebra. This period in geometry was overtaken by research on the general algebraic curve by Clebsch , Riemann , Max Noether and others, which stretched existing techniques, and then by invariant theory. Towards the end of the century, the Italian school of algebraic geometry Enriques , Segre , Severi broke out of the traditional subject matter into an area demanding deeper techniques. During the later part of the 19th century, the detailed study of projective geometry became less fashionable, although the literature is voluminous. Some important work was done in enumerative geometry in particular, by Schubert, that is now considered as anticipating the theory of Chern classes , taken as representing the algebraic topology of Grassmannians. Paul Dirac studied projective geometry and used it as a basis for developing his concepts of quantum mechanics , although his published results were always in algebraic form. See a blog article referring to an article and a book on this subject, also to a talk Dirac gave to a general audience during in Boston about projective geometry, without specifics as to its application in his physics. Description[edit] Projective geometry is less restrictive than either Euclidean geometry or affine geometry. It is an intrinsically non- metrical geometry, meaning that facts are independent of any metric structure. Under the projective transformations, the incidence structure and the relation of projective harmonic conjugates are preserved. A projective range is the one-dimensional foundation. Projective geometry formalizes one of the central principles of perspective art: In essence, a projective geometry may be thought of as an extension of Euclidean geometry in which the "direction" of each line is subsumed within the line as an extra "point", and in which a "horizon" of directions corresponding to coplanar lines is regarded as a "line". Thus, two parallel lines meet on a horizon line by virtue of their incorporating the same direction. Idealized directions are referred to as points at infinity, while idealized horizons are referred to as lines at infinity. In turn, all these lines lie in the plane at infinity. However, infinity is a metric concept, so a purely projective geometry does not single out any points, lines or planes in this regardâ€”those at infinity are treated just like any others. Because a Euclidean geometry is contained within a projective geometryâ€”with projective geometry having a simpler foundationâ€—general results in Euclidean geometry may be derived in a more transparent manner, where separate but similar theorems of Euclidean geometry may be handled collectively within the framework of projective geometry. For example, parallel and nonparallel lines need not be treated as separate cases; rather an arbitrary projective plane is singled out as the ideal plane and located "at infinity" using homogeneous coordinates. But for dimension 2, it must be separately postulated. Projective geometry also includes a full theory of conic sections , a subject also extensively developed in Euclidean geometry. There are advantages to being able to think of a hyperbola and an ellipse as distinguished only by the way the hyperbola lies across the line at infinity; and that a parabola is distinguished only by being tangent to the same line. The whole family of circles can be considered as conics passing through two given points on the line at infinity â€” at the cost of requiring complex coordinates. Since coordinates are not "synthetic", one replaces them by fixing a line and two points on it, and considering the linear system of all conics passing through those points as the basic object of study. This method proved very attractive to talented geometers, and the topic was studied thoroughly. An example of this method is the multi-volume treatise by H. There are many projective geometries, which may be divided into discrete and continuous: The only projective geometry of dimension 0 is a single point. A projective geometry of dimension 1 consists of a single line containing at least 3 points. The geometric construction of arithmetic operations cannot be performed in either of these cases. The Fano plane is the projective plane with the fewest points and lines. According to Greenberg and others, the simplest 2-dimensional projective geometry is the Fano plane , which has 3 points on every line, with 7 points and 7 lines in all, having the following collinearities:

FUNDAMENTAL THEOREM OF PROJECTIVE GEOMETRY 1.4.1. pdf

2: COGNITIVE GEOMETRICS - GEOMETRY: Projective Geometry

The fundamental theorem of projective geometry consists of the three following theorems. Given two projective frames of a projective space P, there is exactly one homography of P that maps the first frame onto the second one.

The French Revolution provoked a radical rethinking of education in France, and mathematics was given a prominent role. The formula in the figure reads k is to l as m is to n if and only if line DE is parallel to line AB. This theorem then enables one to show that the small and large triangles are similar. Now consider the effect produced by projecting these line segments onto another plane as shown in the figure. It was Desargues who first introduced a single point at infinity to represent the projected intersection of parallel lines. Furthermore, he collected all the points along the horizon in one line at infinity. Other properties are preserved, however. For instance, two different points have a unique connecting line, and two different lines have a unique point of intersection. Although almost nothing else seems to be invariant under projective mappings, one should note that lines are mapped onto lines. This means that if three points are collinear share a common line , then the same will be true for their projections. Thus, collinearity is another invariant property. Similarly, if three lines meet in a common point, so will their projections. The following theorem is of fundamental importance for projective geometry. In its first variant , by Pappus of Alexandria fl. The second variant , by Pascal , as shown in the figure, uses certain properties of circles: If the distinct points A, B, C, D, E, and F are on one circle, then the three intersection points x, y, and z defined as above are collinear. There is one more important invariant under projective mappings, known as the cross ratio see the figure. It may also be written as the quotient of two ratios: The latter formulation reveals the cross ratio as a ratio of ratios of distances. However, this result remained a mere curiosity until its real significance became gradually clear in the 19th century as mappings became more and more important for transforming problems from one mathematical domain to another. Projective conic sections Conic sections can be regarded as plane sections of a right circular cone see the figure. Depending on the orientation of the cutting plane, the image of the circle will be a circle , an ellipse , a parabola , or a hyperbola. Conic sectionsThe conic sections result from intersecting a plane with a double cone, as shown in the figure. There are three distinct families of conic sections: Start by selecting six points on a conic section and project them back onto the base circle. As given earlier, the three relevant intersection points for six points on the circle will be collinear. Now project all nine points back to the conic section. Since collinear points the three intersection points from the circle are mapped onto collinear points, the theorem holds for any conic section. In this way the projective point of view unites the three different types of conics. Similarly, more complicated curves and surfaces in higher-dimensional spaces can be unified through projections. For example, Isaac Newton â€” showed that all plane curves defined by polynomials in x and y of degree 3 the highest power of the variables is 3 can be obtained as projective images of just five types of polynomials.

FUNDAMENTAL THEOREM OF PROJECTIVE GEOMETRY 1.4.1. pdf

3: Homography | Revolvy

Fundamental Theorem of Projective Geometry Printout All truths are easy to understand once they are discovered; the point is to discover them.. â€" Galileo Galilei () In the introduction to projective geometry, we stated that in a later section we would consider a mapping between two pencils of points.

The composition of two central collineations, while still a homography in general, is not a central collineation. In fact, every homography is the composition of a finite number of central collineations. In synthetic geometry, this property, which is a part of the fundamental theory of projective geometry is taken as the definition of homographies. These collineations are called automorphic collineations. The fundamental theorem of projective geometry consists of the three following theorems. Given two projective frames of a projective space P, there is exactly one homography of P that maps the first frame onto the second one. If the dimension of a projective space P is at least two, every collineation of P is the composition of an automorphic collineation and a homography. In particular, over the reals, every collineation of a projective space of dimension at least two is a homography. In particular, if the dimension of the implied projective space is at least two, every homography is the composition of a finite number of central collineations. If projective spaces are defined by means of axioms synthetic geometry , the third part is simply a definition. On the other hand, if projective spaces are defined by means of linear algebra , the first part is an easy corollary of the definitions. Therefore, the proof of the first part in synthetic geometry, and the proof of the third part in terms of linear algebra both are fundamental steps of the proof of the equivalence of the two ways of defining projective spaces. Homography groups[edit] As every homography has an inverse mapping and the composition of two homographies is another, the homographies of a given projective space form a group. As all the projective spaces of the same dimension over the same field are isomorphic, the same is true for their homography groups. They are therefore considered as a single group acting on several spaces, and only the dimension and the field appear in the notation, not the specific projective space. For example, PGL 2, 7 acts on the eight points in the projective line over the finite field GF 7 , while PGL 2, 4 , which is isomorphic to the alternating group A5, is the homography group of the projective line with five points. When the points and lines of the projective space are viewed as a block design , whose blocks are the sets of points contained in a line, it is common to call the collineation group the automorphism group of the design. Cross-ratio[edit] Use of cross-ratios in projective geometry to measure real-world dimensions of features depicted in a perspective projection. In 1 , the width of the side street, W is computed from the known widths of the adjacent shops. In 2 , the width of only one shop is needed because a vanishing point , V is visible. Cross-ratio The cross-ratio of four collinear points is an invariant under the homography that is fundamental for the study of the homographies of the lines. Three distinct points a, b and c on a projective line over a field F form a projective frame of this line. In other words, if d has homogeneous coordinates [k: Homographies act on a projective line over A, written P A , consisting of points U a, b with homogeneous coordinates. The homographies on P A are described by matrix mappings U.

FUNDAMENTAL THEOREM OF PROJECTIVE GEOMETRY 1.4.1. pdf

4: Fundamental Theorem

and Projective Geometry have combined giving a completely new perspective on Geometry as a whole. It is in human nature, however, to consider what other loci there may be, and this paper turns to an interesting type of locus, the cubic.

Projective geometry is concerned with incidences, that is, where elements such as lines planes and points either coincide or not. The converse is true i. This illustrates a fact about incidences and has nothing to say about measurements. This is characteristic of pure projective geometry. The most fundamental fact is that there is one and only one line joining two distinct points in a plane, and dually two lines meet in one and only one point. But what, you may ask, about parallel lines? There is just one ideal point associated with each direction in the plane, in which all parallel lines in such a direction meet. The next figure shows the process of projecting a RANGE of points on a yellow line into another range on a distinct blue line. The points are indicated by the centre points of white crosses. Projection and section are dual processes. The above procedure may be repeated for a sequence of projections and sections. These are the shapes arising if a plane cuts a cone, and in fact include a pair of straight lines and also, of course, the circle. Using the dual process a conic can be constructed by points using projective pencils. There are many theorems that there is no space to explain here. A particularly important subject for counter space is that of polarity , which is related to the principle of duality. This is illustrated below. The fact to note here is that the polars of the points on a line form a pencil in a point, which is the polar of that line. The situation is self-dual. In three dimensions we illustrate the same principle but with a sphere and a point. The cone with its apex in that point, and which is tangential to the sphere, determines a plane red containing the circle of contact. Similarly to the two-dimensional case, if we take the polar planes of all the points in a plane, they all contain a common point which is the pole of that plane. Lines are now self-polar. When counter space is studied this property of points and planes is used to conceptualise the idea of a negative space, as we reverse the roles of centre and infinity. In a plane the ideal points form an ideal line, and in space they form an ideal plane or plane at infinity. A special case of projective geometry can be defined which leaves the plane at infinity invariant as a whole i. This is known as affine geometry. A further special case is possible where the volume of objects remains invariant, which is known as special affine geometry. Finally a further specialisation ensures that lengths and angles are invariant, which is metric geometry, so called because measurements remain unaltered by its transformations. The cross ratio of four points is the only numerical invariant of projective geometry if it can be related to Euclidean space. Flat line pencils and axial pencils of planes containing a common line also have cross ratios. If two quadrangles have 5 pairs of corresponding sides meeting in collinear points, the sixth pair meet on the same line. Construction of two pairs of points harmonically separated, which have a cross ratio of A basic projective transformation in which corresponding sides meet on a fixed line called the axis, and corresponding points lie on lines through the centre. They show repeated transformations of the points on a line. Breathing or hyperbolic Measure. A point is shown moving along a line between two invariant points with construction. Growth Measure with one invariant point at infinity. The ratios of the distances of successive points from the other invariant point are constant. Step or parabolic Measure, in which the two invariant points coincide. This is how equal steps appear in counter space for our ordinary consciousness. Step Measure with both invariant points at infinity, which yields equal steps. The proof follows from the fact that triangles on the same base and between the same parallels are equal in area. Circling or elliptic Measure in which there are no invariant points. The two auxiliary lines used in the above constructions may be regarded as special cases of a conic. If you attempt to impose three invariant points on a line e. This is the Fixed Point Theorem of projective geometry. The following animations show the application of the above to transformation of a plane, in these examples lines being transformed by means of two measures on two sides of the invariant triangle. Projective transformation in which it is demonstrated that parallelism is not conserved. Affine transformation where two red parallel lines are transformed into two parallel lines one green and one blue. This is affine because one side of the invariant triangle is at infinity since each measure has an invariant point at infinity. References 6 and 8 and 9 give a good introduction to projective geometry, where the above facts are proved.

5: Projective geometry | www.enganchecubano.com

The Fundamental Theorem of Projective Geometry A projectivity is completely determined when three collinear points and the corresponding collinear points are given. You can replace either one or both of the "three collinear points" with "three concurrent lines" and still have a true statement.

The composition of two central collineations, while still a homography in general, is not a central collineation. In fact, every homography is the composition of a finite number of central collineations. In synthetic geometry, this property, which is a part of the fundamental theory of projective geometry is taken as the definition of homographies. These collineations are called automorphic collineations. The fundamental theorem of projective geometry consists of the three following theorems. Given two projective frames of a projective space P, there is exactly one homography of P that maps the first frame onto the second one. If the dimension of a projective space P is at least two, every collineation of P is the composition of an automorphic collineation and a homography. In particular, over the reals, every collineation of a projective space of dimension at least two is a homography. In particular, if the dimension of the implied projective space is at least two, every homography is the composition of a finite number of central collineations. If projective spaces are defined by means of axioms synthetic geometry , the third part is simply a definition. On the other hand, if projective spaces are defined by means of linear algebra , the first part is an easy corollary of the definitions. Therefore, the proof of the first part in synthetic geometry, and the proof of the third part in terms of linear algebra both are fundamental steps of the proof of the equivalence of the two ways of defining projective spaces. Homography groups As every homography has an inverse mapping and the composition of two homographies is another, the homographies of a given projective space form a group. As all the projective spaces of the same dimension over the same field are isomorphic, the same is true for their homography groups. They are therefore considered as a single group acting on several spaces, and only the dimension and the field appear in the notation, not the specific projective space. For example, PGL 2, 7 acts on the eight points in the projective line over the finite field GF 7 , while PGL 2, 4 , which is isomorphic to the alternating group A, is the homography group of the projective line with five points. When the points and lines of the projective space are viewed as a block design , whose blocks are the sets of points contained in a line, it is common to call the collineation group the automorphism group of the design. Cross-ratio Use of cross-ratios in projective geometry to measure real-world dimensions of features depicted in a perspective projection. In 1 , the width of the side street, W is computed from the known widths of the adjacent shops. In 2 , the width of only one shop is needed because a vanishing point , V is visible. The cross-ratio of four collinear points is an invariant under the homography that is fundamental for the study of the homographies of the lines. Three distinct points a, b and c on a projective line over a field F form a projective frame of this line. In other words, if d has homogeneous coordinates [k: Homographies act on a projective line over A, written P A , consisting of points U a, b with homogeneous coordinates. The homographies on P A are described by matrix mappings U.

6: Projective geometry - Wikipedia

fundamental theorem of projective geometry Theorem 1 (Fundamental Theorem of Projective Geometry I). Every bijective order-preserving map (projectivity) f: P â•¢ G â•¢ (V) â†' P â•¢ G â•¢ (W), where V and W are vector spaces of finite dimension not equal to 2, is induced by a semilinear transformation f ^: V â†' W.

7: fundamental theorem of projective geometry

Abstract. The following version of the fundamental theorem is proved: Let V, W be vector spaces and g: P(V)\E â†' P(W) a morphism between the associated projective www.enganchecubano.com the image of g is not contained in a line, then there exists a semilinear map f: V â†' W which induces g.

8: Homography - Wikipedia

This is the introductory video to my series on the foundations of projective geometry. This course explains projective geometry in a mathematically rigorous way, using axioms and algebra, although.

9: Projective Geometry - GeoGebraWiki

The fundamental theorem of projective geometry states that any four planar non-collinear points (a quadrangle) can be sent to any quadrangle via a projectivity, that is a sequence of perspectivities.

FUNDAMENTAL THEOREM OF PROJECTIVE GEOMETRY 1.4.1. pdf

Introduction to english phonetics Young peoples history of Illinois Siberian transfer Verizon data breach investigations report 2017 Global change-what you can do Epilogue, Thinking with Tocqueville. Philip W. Silver. American business tokens Electric machinery and power system fundamentals 3rd edition Bears Big Blue House Painting, staining, and finishing Breath Prayers for Women Mad hatter wonderland sheet music Sociology in modules The Republic of Korea, employment, industrialisation and trade Young Love-Old Secrets Choices From My Heart Solar cells and optics for photovoltaic concentration From his appointment as a missionary, to his commencing his mission among the Indians at Kaunaumeek, in N Java game tutorial for beginners Jubilee of the world Statistics for engineers and scientists 4th edition solutions Life path number : what each life path offers and desires in love Outer Banks Low Price Daily problems and weekly puzzlers Sketches of frontier and Indian life on the upper Missouri and great plains. Can i my files on kindle The political animal jeremy paxman Overrunning Germany Early Dutch, German English Printers Marks: By J. Ph. Berjeau. Incident at Finnegans wake. Electrochemically Fabricated Nanometer-sized Molecular Junctions Saddle Tramp Marshal Model description Hunger games 1 The Gulf in Turmoil X Isle (X Isle Mini Ser.) Introduction to Philosophy and Ethics (Access to Philosophy) Tangipahoa and the forced migration A collection of hieroglyphs