MATHEMATICAL MODELING OF DIVERSE PHENOMENA pdf

1: Scientific modelling - Wikipedia

This book is intended for those students, engineers, scientists, and applied mathematicians who find it necessary to formulate models of diverse phenomena. To facilitate the formulation of such models, some aspects of the tensor calculus will be introduced.

Classifications[edit] Mathematical models are usually composed of relationships and variables. Relationships can be described by operators , such as algebraic operators, functions, differential operators, etc. Variables are abstractions of system parameters of interest, that can be quantified. Several classification criteria can be used for mathematical models according to their structure: If all the operators in a mathematical model exhibit linearity , the resulting mathematical model is defined as linear. A model is considered to be nonlinear otherwise. The definition of linearity and nonlinearity is dependent on context, and linear models may have nonlinear expressions in them. For example, in a statistical linear model , it is assumed that a relationship is linear in the parameters, but it may be nonlinear in the predictor variables. Similarly, a differential equation is said to be linear if it can be written with linear differential operators , but it can still have nonlinear expressions in it. In a mathematical programming model, if the objective functions and constraints are represented entirely by linear equations , then the model is regarded as a linear model. If one or more of the objective functions or constraints are represented with a nonlinear equation, then the model is known as a nonlinear model. Nonlinearity, even in fairly simple systems, is often associated with phenomena such as chaos and irreversibility. Although there are exceptions, nonlinear systems and models tend to be more difficult to study than linear ones. A common approach to nonlinear problems is linearization , but this can be problematic if one is trying to study aspects such as irreversibility, which are strongly tied to nonlinearity. A dynamic model accounts for time-dependent changes in the state of the system, while a static or steady-state model calculates the system in equilibrium, and thus is time-invariant. Dynamic models typically are represented by differential equations or difference equations. If all of the input parameters of the overall model are known, and the output parameters can be calculated by a finite series of computations, the model is said to be explicit. In such a case the model is said to be implicit. A discrete model treats objects as discrete, such as the particles in a molecular model or the states in a statistical model ; while a continuous model represents the objects in a continuous manner, such as the velocity field of fluid in pipe flows, temperatures and stresses in a solid, and electric field that applies continuously over the entire model due to a point charge. A deterministic model is one in which every set of variable states is uniquely determined by parameters in the model and by sets of previous states of these variables; therefore, a deterministic model always performs the same way for a given set of initial conditions. Conversely, in a stochastic modelâ€”usually called a " statistical model "â€”randomness is present, and variable states are not described by unique values, but rather by probability distributions. Deductive, inductive, or floating: A deductive model is a logical structure based on a theory. An inductive model arises from empirical findings and generalization from them. The floating model rests on neither theory nor observation, but is merely the invocation of expected structure. Application of mathematics in social sciences outside of economics has been criticized for unfounded models. Physical theories are almost invariably expressed using mathematical models. Throughout history, more and more accurate mathematical models have been developed. It is possible to obtain the less accurate models in appropriate limits, for example relativistic mechanics reduces to Newtonian mechanics at speeds much less than the speed of light. Quantum mechanics reduces to classical physics when the quantum numbers are high. For example, the de Broglie wavelength of a tennis ball is insignificantly small, so classical physics is a good approximation to use in this case. It is common to use idealized models in physics to simplify things. Massless ropes, point particles, ideal gases and the particle in a box are among the many simplified models used in physics. These laws are such as a basis for making mathematical models of real situations. Many real situations are very complex and thus modeled approximate on a computer, a model that is computationally feasible to compute is made from the basic laws or from approximate models made from the basic laws. In engineering , physics models are often made by mathematical methods such as finite element analysis. Different mathematical models use different geometries that are not necessarily accurate descriptions of the geometry of the universe. Euclidean geometry is much used in classical physics, while special relativity and general relativity are examples of theories that use geometries which are not Euclidean. Some applications[edit] Since prehistorical times simple models such as maps and diagrams have been used. Often when engineers analyze a system to be controlled or optimized, they use a mathematical model. In analysis, engineers can build a descriptive model of the system as a hypothesis of how the system could work, or try to estimate how an unforeseeable event could affect the system. Similarly, in control of a system, engineers can try out different control approaches in simulations. A mathematical model usually describes a system by a set of variables and a set of equations that establish relationships between the variables. Variables may be of many types; real or integer numbers, boolean values or strings , for example. The actual model is the set of functions that describe the relations between the different variables. Building blocks[edit] In business and engineering , mathematical models may be used to maximize a certain output. The system under consideration will require certain inputs. The system relating inputs to outputs depends on other variables too: Decision variables are sometimes known as independent variables. Exogenous variables are sometimes known as parameters or constants. The variables are not independent of each other as the state variables are dependent on the decision, input, random, and exogenous variables. Furthermore, the output variables are dependent on the state of the system represented by the state variables. Objectives and constraints of the system and its users can be represented as functions of the output variables or state variables. Depending on the context, an objective function is also known as an index of performance, as it is some measure of interest to the user. Although there is no limit to the number of objective functions and constraints a model can have, using or optimizing the model becomes more involved computationally as the number increases. For example, economists often apply linear algebra when using input-output models. Complicated mathematical models that have many variables may be consolidated by use of vectors where one symbol represents several variables. The usual representation of this black box system is a data flow diagram centered in the box. Mathematical modeling problems are often classified into black box or white box models, according to how much a priori information on the system is available. A black-box model is a system of which there is no a priori information available. A white-box model also called glass box or clear box is a system where all necessary information is available. Practically all systems are somewhere between the black-box and white-box models, so this concept is useful only as an intuitive guide for deciding which approach to take. Usually it is preferable to use as much a priori information as possible to make the model more accurate. Therefore, the white-box models are usually considered easier, because if you have used the information correctly, then the model will behave correctly. Often the a priori information comes in forms of knowing the type of functions relating different variables. For example, if we make a model of how a medicine works in a human system, we know that usually the amount of medicine in the blood is an exponentially decaying function. But we are still left with several unknown parameters; how rapidly does the medicine amount decay, and what is the initial amount of medicine in blood? This example is therefore not a completely white-box model. These parameters have to be estimated through some means before one can use the model. In black-box models one tries to estimate both the functional form of relations between variables and the numerical parameters in those functions. Using a priori information we could end up, for example, with a set of functions that probably could describe the system adequately. If there is no a priori information we would try to use functions as general as possible to cover all different models. An often used approach for black-box models are neural networks which usually do not make assumptions about incoming data. Alternatively the NARMAX Nonlinear AutoRegressive Moving Average model with eXogenous inputs algorithms which were developed as part of nonlinear system identification [3] can be used to select the model terms, determine the model structure, and estimate the unknown parameters in the presence of correlated and nonlinear noise. The advantage of NARMAX models compared to neural networks is that NARMAX produces models that can be written down and related to the underlying process, whereas neural networks produce an approximation that is opaque. Subjective information[edit] Sometimes it is useful to incorporate subjective information into a mathematical model. This can be done based on intuition , experience , or expert opinion , or based on convenience of mathematical form. Bayesian statistics provides a theoretical framework for incorporating such subjectivity into a rigorous analysis: An example of when such approach would be necessary is a situation in which an experimenter bends a coin slightly and tosses it once, recording whether it comes up heads, and is then given the task of predicting the probability that the next flip comes up heads. After bending the coin, the true probability that the coin will come up heads is unknown; so the experimenter would need to make a decision perhaps by looking at the shape of the coin about what prior distribution to use. Incorporation of such subjective information might be important to get an accurate estimate of the probability. Complexity[edit] In general, model complexity involves a trade-off between simplicity and accuracy of the model. While added complexity usually improves the realism of a model, it can make the model difficult to understand and analyze, and can also pose computational problems, including numerical instability. Thomas Kuhn argues that as science progresses, explanations tend to become more complex before a paradigm shift offers radical simplification[citation needed]. For example, when modeling the flight of an aircraft, we could embed each mechanical part of the aircraft into our model and would thus acquire an almost white-box model of the system. However, the computational cost of adding such a huge amount of detail would effectively inhibit the usage of such a model. Additionally, the uncertainty would increase due to an overly complex system, because each separate part induces some amount of variance into the model. It is therefore usually appropriate to make some approximations to reduce the model to a sensible size. Engineers often can accept some approximations in order to get a more robust and simple model. Training and tuning[edit] Any model which is not pure white-box contains some parameters that can be used to fit the model to the system it is intended to describe. If the modeling is done by an artificial neural network or other machine learning , the optimization of parameters is called training[citation needed][why? In more conventional modeling through explicitly given mathematical functions, parameters are often determined by curve fitting [citation needed]. Model evaluation[edit] A crucial part of the modeling process is the evaluation of whether or not a given mathematical model describes a system accurately. This question can be difficult to answer as it involves several different types of evaluation. Fit to empirical data[edit] Usually the easiest part of model evaluation is checking whether a model fits experimental measurements or other empirical data. In models with parameters, a common approach to test this fit is to split the data into two disjoint subsets: The training data are used to estimate the model parameters. This practice is referred to as cross-validation in statistics. Defining a metric to measure distances between observed and predicted data is a useful tool of assessing model fit.

2: Mathematical model - Wikipedia

Tensor calculus is applied to the formulation of mathematical models of diverse phenomena. Aeronautics, fluid dynamics, and cosmology are among the areas of application. The feasibility of combining tensor methods and computer capability to formulate problems is demonstrated.

Overview[edit] A scientific model seeks to represent empirical objects, phenomena, and physical processes in a logical and objective way. All models are in simulacra, that is, simplified reflections of reality that, despite being approximations, can be extremely useful. Complete and true representation may be impossible, but scientific debate often concerns which is the better model for a given task, e. The aim of these attempts is to construct a formal system that will not produce theoretical consequences that are contrary to what is found in reality. Predictions or other statements drawn from such a formal system mirror or map the real world only insofar as these scientific models are true. Such computer models are in silico. Other types of scientific models are in vivo living models, such as laboratory rats and in vitro in glassware, such as tissue culture. Direct measurement of outcomes under controlled conditions see Scientific method will always be more reliable than modelled estimates of outcomes. Within modelling and simulation , a model is a task-driven, purposeful simplification and abstraction of a perception of reality, shaped by physical, legal, and cognitive constraints. Simplifications leave all the known and observed entities and their relation out that are not important for the task. Abstraction aggregates information that is important, but not needed in the same detail as the object of interest. Both activities, simplification and abstraction, are done purposefully. However, they are done based on a perception of reality. This perception is already a model in itself, as it comes with a physical constraint. There are also constraints on what we are able to legally observe with our current tools and methods, and cognitive constraints which limit what we are able to explain with our current theories. This model comprises the concepts, their behavior, and their relations in formal form and is often referred to as a conceptual model. In order to execute the model, it needs to be implemented as a computer simulation. This requires more choices, such as numerical approximations or the use of heuristics. A steady state simulation provides information about the system at a specific instant in time usually at equilibrium, if such a state exists. A dynamic simulation provides information over time. A simulation brings a model to life and shows how a particular object or phenomenon will behave. Such a simulation can be useful for testing, analysis, or training in those cases where real-world systems or concepts can be represented by models. In general, a system is a construct or collection of different elements that together can produce results not obtainable by the elements alone. There are two types of system models: Typically a model will deal with only some aspects of the phenomenon in question, and two models of the same phenomenon may be essentially differentâ€"that is to say, that the differences between them comprise more than just a simple renaming of components. In any case, users of a model need to understand the assumptions made that are pertinent to its validity for a given use. Building a model requires abstraction. Assumptions are used in modelling in order to specify the domain of application of the model. For example, the special theory of relativity assumes an inertial frame of reference. This assumption was contextualized and further explained by the general theory of relativity. A model makes accurate predictions when its assumptions are valid, and might well not make accurate predictions when its assumptions do not hold. Such assumptions are often the point with which older theories are succeeded by new ones the general theory of relativity works in non-inertial reference frames as well. The term "assumption" is actually broader than its standard use, etymologically speaking. The Oxford English Dictionary OED and online Wiktionary indicate its Latin source as assumere "accept, to take to oneself, adopt, usurp" , which is a conjunction of ad- "to, towards, at" and sumere to take. The root survives, with shifted meanings, in the Italian sumere and Spanish sumir. One way to modify the model is by restricting the domain over which it is credited with having high validity. A case in point is Newtonian physics, which is highly useful except for the very small, the very fast, and the very massive phenomena of the universe. However, a fit to empirical data alone is not sufficient for a model to be accepted as valid. Other factors important in evaluating a model include: Visualization[edit] Visualization is any technique for creating images, diagrams, or animations to communicate a message. Visualization through visual imagery has been an effective way to communicate both abstract and concrete ideas since the dawn of man. Space mapping[edit] Space mapping refers to a methodology that employs a "quasi-global" modeling formulation to link companion "coarse" ideal or low-fidelity with "fine" practical or high-fidelity models of different complexities. In engineering optimization , space mapping aligns maps a very fast coarse model with its related expensive-to-compute fine model so as to avoid direct expensive optimization of the fine model. The alignment process iteratively refines a "mapped" coarse model surrogate model. Types of scientific modelling[edit].

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3: Professor Uses Language of Math to Describe Natural Phenomena - UT Dallas News

The book is concerned with the formulation of models of diverse phenomena. To facilitate the formulation of such models, some aspects of the tensor calculus that are considered necessary for an.

Print this page Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data. A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situationsâ€"modeling a delivery route, a production schedule, or a comparison of loan amortizationsâ€"need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity. Some examples of such situations might include: Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed. Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player. Designing the layout of the stalls in a school fair so as to raise as much money as possible. Analyzing stopping distance for a car. Modeling savings account balance, bacterial colony growth, or investment growth. Engaging in critical path analysis, e. Analyzing risk in situations such as extreme sports, pandemics, and terrorism. Relating population statistics to individual predictions. In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations. One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function. The basic modeling cycle is summarized in the diagram. It involves 1 identifying variables in the situation and selecting those that represent essential features, 2 formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, 3 analyzing and performing operations on these relationships to draw conclusions, 4 interpreting the results of the mathematics in terms of the original situation, 5 validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, 6 reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle. In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive modelâ€"for example, graphs of global temperature and atmospheric CO2 over time. Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies until cut-off mechanisms such as pollution or starvation intervene follows from a constant reproduction rate. Functions are an important tool for analyzing such problems. Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena e. Modeling Standards Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards.

4: High School: Modeling | Common Core State Standards Initiative

Mathematical models Calculus of tensors This book is intended for those students, engineers, scientists, and applied mathematicians who find it necessary to formulate models of diverse phenomena.

5: Modeling Natural Disasters with Mathematical Functions

This book is intended for those students, engineers, scientists, and applied mathematicians who find it necessary to formulate models of diverse phenomena.

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