

## 1: Numerical analysis - Wikipedia

*Numerical mathematics is a vast field whose importance cannot be over-emphasized. The solution of real-life problems quite often can't be achieved without resorting to the methods of numerical mathematics.*

Much effort has been put in the development of methods for solving systems of linear equations. Standard direct methods, i. Iterative methods such as the Jacobi method , Gauss-Seidel method , successive over-relaxation and conjugate gradient method are usually preferred for large systems. General iterative methods can be developed using a matrix splitting. Root-finding algorithms are used to solve nonlinear equations they are so named since a root of a function is an argument for which the function yields zero. Linearization is another technique for solving nonlinear equations. Solving eigenvalue or singular value problems[ edit ] Several important problems can be phrased in terms of eigenvalue decompositions or singular value decompositions. For instance, the spectral image compression algorithm [4] is based on the singular value decomposition. The corresponding tool in statistics is called principal component analysis. Mathematical optimization Optimization problems ask for the point at which a given function is maximized or minimized. Often, the point also has to satisfy some constraints. The field of optimization is further split in several subfields, depending on the form of the objective function and the constraint. For instance, linear programming deals with the case that both the objective function and the constraints are linear. A famous method in linear programming is the simplex method. The method of Lagrange multipliers can be used to reduce optimization problems with constraints to unconstrained optimization problems. Numerical integration Numerical integration, in some instances also known as numerical quadrature , asks for the value of a definite integral. These methods rely on a "divide and conquer" strategy, whereby an integral on a relatively large set is broken down into integrals on smaller sets. In higher dimensions, where these methods become prohibitively expensive in terms of computational effort, one may use Monte Carlo or quasi-Monte Carlo methods see Monte Carlo integration , or, in modestly large dimensions, the method of sparse grids. Numerical ordinary differential equations and Numerical partial differential equations Numerical analysis is also concerned with computing in an approximate way the solution of differential equations , both ordinary differential equations and partial differential equations. Partial differential equations are solved by first discretizing the equation, bringing it into a finite-dimensional subspace. This can be done by a finite element method , a finite difference method, or particularly in engineering a finite volume method. The theoretical justification of these methods often involves theorems from functional analysis. This reduces the problem to the solution of an algebraic equation. List of numerical analysis software and Comparison of numerical analysis software Since the late twentieth century, most algorithms are implemented in a variety of programming languages. The Netlib repository contains various collections of software routines for numerical problems, mostly in Fortran and C. Also, any spreadsheet software can be used to solve simple problems relating to numerical analysis.

*Numerical mathematics is the branch of mathematics that proposes, develops, analyzes and applies methods from scientific computing to several fields including analysis, linear algebra, geometry, approximation theory, functional equations, optimization and differential equations.*

It shows that R is a viable computing environment for implementing and applying numerical methods, also outside the realm of statistics. The task view will not cover differential equations, optimization problems and solvers, or packages and functions operating on times series, because all these topics are treated extensively in the corresponding task views DifferentialEquations , Optimization , and TimeSeries. All these task views together will provide a good selection of what is available in R for the area of numerical mathematics. The HighPerformanceComputing task view with its many links for parallel computing may also be of interest. The task view has been created to provide an overview of the topic. If some packages are missing or certain topics in numerical math should be treated in more detail, please let the maintainer know. Numerical Linear Algebra As statistics is based to a large extent on linear algebra, many numerical linear algebra routines are present in R, and some only implicitly. The recommended package Matrix provides classes and methods for dense and sparse matrices and operations on them, for example Cholesky and Schur decomposition, matrix exponential, or norms and conditional numbers for sparse matrices. SparseM provides classes and methods for sparse matrices and for solving linear and least-squares problems in sparse linear algebra Package rmumps provides a wrapper for the MUMPS library, solving large linear systems of equations applying a parallel sparse direct solver Rlinsolve is a collection of iterative solvers for sparse linear system of equations. Stationary iterative solvers such as Jacobi or Gauss-Seidel, as well as nonstationary Krylov subspace methods are provided. The packages geigen and QZ compute generalized eigenvalues and -vectors for pairs of matrices, and QZ generalized Schur decompositions. Package onion contains routines for manipulating quaternions and octonians normed division algebras over the real numbers ; quaternions can be useful for handling rotations in three-dimensional space. Special Functions Many special mathematical functions are present in R, especially logarithms and exponentials, trigonometric and hyperbolic functions, or Bessel and Gamma functions. Many more special functions are available in contributed packages. Airy and Bessel functions, for real and complex numbers, are also computed in package Bessel , with approximations for large arguments. Package pracma includes special functions, such as error functions and inverses, incomplete and complex gamma function, exponential and logarithmic integrals, Fresnel integrals, the polygamma and the Dirichlet and Riemann zeta functions. The hypergeometric and generalized hypergeometric function, is computed in hypergeo , including transformation formulas and special values of the parameters. There are tools for visualizing complex functions. Package expint wraps C-functions from the GNU Scientific Library to calculate exponential integrals and the incomplete Gamma function, including negative values for its first argument. Package lamW implements both real-valued branches of the Lambert W function using Rcpp. Polynomials Function polyroot in base R determines all zeros of a polynomial, based on the Jenkins-Traub algorithm. Packages polynom and PolynomF provide similar functionality for manipulating univariate polynomials, like evaluating polynomials Horner scheme , differentiating or integrating them, or solving polynomials, i. Package MonoPoly fits univariate polynomials to given data, applying different algorithms. For multivariate polynomials, package multipol provides various tools to manipulate and combine these polynomials of several variables. Package mpoly facilitates symbolic manipulations on multivariate polynomials, including basic differential calculus operations on polynomials, plus some Groebner basis calculations. Package orthopolynom consists of a collection of functions to construct orthogonal polynomials and their recurrence relations, among them Chebyshev, Hermite, and Legendre polynomials, as well as spherical and ultraspherical polynomials. There are functions to operate on these polynomials. Differentiation and Integration D and deriv in base R compute derivatives of simple expressions symbolically. Function integrate implements an approach for numerically integrating univariate functions in R. It applies adaptive Gauss-Kronrod quadrature and can handle singularities and unbounded domains to a certain extent. Package Deriv provides an extended solution for

symbolic differentiation in R; the user can add custom derivative rules, and the output for a function will be an executable function again. Works only with Julia v0. Package `gaussquad` contains a collection of functions to perform Gaussian quadrature, among them Chebyshev, Hermite, Laguerre, and Legendre quadrature rules, explicitly returning nodes and weights in each case. Function `gaussquad` in package `statmod` does a similar job. Adaptive multivariate integration over hyper-rectangles in  $n$ -dimensional space is available in package `cubature` as function `adaptIntegrate`, based on a C library of the same name. The integrand functions can even be multi-valued. These grids will be based on different quadrature rules such as Newton-Cotes or Gauss quadrature formulas. Package `SparseGrid` provides another approach to multivariate integration in high-dimensional spaces. It creates sparse  $n$ -dimensional grids that can be used as with quadrature rules. Package `SphericalCubature` employs cubature to integrate functions over unit spheres and balls in  $n$ -dimensional space; `SimplicialCubature` provides methods to integrate functions over  $m$ -dimensional simplices in  $n$ -dimensional space. Both packages comprise exact methods for polynomials. Package `polyCub` holds some routines for numerical integration over polygonal domains in two dimensions. Package `Pade` calculates the numerator and denominator coefficients of the Pade approximation, given the Taylor series coefficients of sufficient length. Interpolation and Approximation Base R provides functions `approx` for constant and linear interpolation, and `spline` for cubic Hermite spline interpolation, while `smooth`. Base package `splines` creates periodic interpolation splines in function `periodicSpline`. Interpolation of irregularly spaced data is possible with the `akima` package: This package is distributed under ACM license and not available for commercial use. Package `signal` contains several filters to smooth discrete data, notably `interp1` for linear, spline, and cubic interpolation, `pchip` for piecewise cubic Hermite interpolation, and `sgolay` for Savitzky-Golay smoothing. Package `pracma` provides barycentric Lagrange interpolation in 1 and 2 dimensions in `barylag` resp. The `interp` package provides bivariate data interpolation on regular and irregular grids, either linear or using splines. Currently the piecewise linear interpolation part is implemented. It is intended to provide a free replacement for the ACM licensed `akima`: Package `chebpol` contains methods for creating multivariate Chebyshev and multilinear interpolation on regular grids, e. The interpolating function will be monotone in regions where the specified points change monotonically. `Schumaker` in package `schumaker` implements shape-preserving splines, guaranteed to be monotonic resp. `ADPF` uses least-squares polynomial regression and statistical testing to improve Savitzky-Golay smoothing. Package `conicfit` provides several geometric and algebraic algorithms for fitting circles, ellipses, and conics in general. Root Finding and Fixed Points `uniroot`, implementing the Brent-Decker algorithm, is the basic routine in R to find roots of univariate functions. There are implementations of the bisection algorithm in several contributed packages. For root finding with higher precision there is function `unirootR` in the multi-precision package `Rmpfr`. And for finding roots of multivariate functions see the following two packages: For solving nonlinear systems of equations the `BB` package provides non-monotone Barzilai-Borwein spectral methods in `sane`, including a derivative-free variant in `dfsane`, and multi-start features with sensitivity analysis. Package `nleqslv` solves nonlinear systems of equations using alternatively the Broyden or Newton method, supported by strategies such as line searches or trust regions. Package `FixedPoint` provides algorithms for finding fixed point vectors. These algorithms include Anderson acceleration, epsilon extrapolation methods, and minimal polynomial methods. Discrete Mathematics and Number Theory Not so many functions are available for computational number theory. Package `numbers` provides functions for factorization, prime numbers, twin primes, primitive roots, modular inverses, extended GCD, etc. Package `freegroup` provides functionality for manipulating elements of a free group including juxtaposition, inversion, multiplication by a scalar, power operations, and Tietze forms. The `partitions` package enumerates additive partitions of integers, including restricted and unequal partitions. Package `combinat` generates all permutations or all combinations of a certain length of a set of elements  $i$ . Package `arrangements` provides generators and iterators for permutations, combinations and partitions. The iterators allow users to generate arrangements in a fast and memory efficient manner. `RcppAlgos` provides flexible functions for generating combinations or permutations of a vector with or without constraints. The extension package `bigIntegerAlgos` features a quadratic sieve algorithm for completely factoring large integers. Examples are factorization of integers, a probabilistic prime number test,

or operations on big rationals -- for which linear systems of equations can be solved. Special numbers and some special functions are included, as well as routines for root finding, integration, and optimization in arbitrary precision. Brobdingnag handles very large numbers by holding their logarithm plus a flag indicating their sign. An excellent vignette explains how this is done using S4 methods. VeryLargeIntegers implements a multi-precision library that allows to store and manage arbitrarily big integers; it includes probabilistic primality tests and factorization algorithms. It supports symbolic and arbitrary precision computations in calculus and linear algebra. It supports arbitrary precision computations, linear algebra and calculus, solving equations, discrete mathematics, and much more. When calling Python in R data types are automatically converted to their equivalent Python types; when values are returned from Python to R they are converted back to R types. This package from the RStudio team is a kind of standard for calling Python from R. PythonInR is another package to interact with Python from within R. It provides Python classes for vectors, matrices and data. This storage format can also be accessed in Python, Julia, or Scala. This R process can run on a remote machine, variable access and function calls will be delegated through the network. SageMath can be downloaded or used through a Web interface at CoCalc. Julia is "a high-level, high-performance dynamic programming language for numerical computing", which makes it interesting for optimization problems and other demanding scientific computations in R. JuliaCall provides seamless integration between R and Julia. Using the high-level interface, the user can call any Julia function just like an R function with automatic type conversion. Package m2r provides a persistent interface to Macauley2, an extended software program supporting research in algebraic geometry and commutative algebra. Macauley2 has to be installed independently, otherwise a Macauley2 process in the cloud will be instantiated.

## 3: Journal of Numerical Mathematics

*Numerical analysis is the study of algorithms that use numerical approximation (as opposed to general symbolic manipulations) for the problems of mathematical analysis (as distinguished from discrete mathematics).*

The mathematical methods needed for computations in engineering and the sciences must be transformed from the continuous to the discrete in order to be carried out on a computer. For example, the computer integration of a function over an interval is accomplished not by a single sum but by a sequence of small sums. Common perspectives in numerical analysis

Numerical analysis is concerned with all aspects of the numerical solution of a problem, from the theoretical development and understanding of numerical methods to their practical implementation as reliable and efficient computer programs. Most numerical analysts specialize in small subfields, but they share some common concerns, perspectives, and mathematical methods of analysis. These include the following: Examples are the use of interpolation in developing numerical integration methods and root-finding methods. There is widespread use of the language and results of linear algebra, real analysis, and functional analysis with its simplifying notation of norms, vector spaces, and operators. There is a fundamental concern with error, its size, and its analytic form. When approximating a problem, it is prudent to understand the nature of the error in the computed solution. Moreover, understanding the form of the error allows creation of extrapolation processes to improve the convergence behaviour of the numerical method. Numerical analysts are concerned with stability, a concept referring to the sensitivity of the solution of a problem to small changes in the data or the parameters of the problem. Consider the following example. Such a polynomial  $p(x)$  is called unstable or ill-conditioned with respect to the root-finding problem. Numerical methods for solving problems should be no more sensitive to changes in the data than the original problem to be solved. Moreover, the formulation of the original problem should be stable or well-conditioned. Numerical analysts are very interested in the effects of using finite precision computer arithmetic. This is especially important in numerical linear algebra, as large problems contain many rounding errors. Numerical analysts would want to know how this method compares with other methods for solving the problem. Modern applications and computer software

Numerical analysis and mathematical modeling are essential in many areas of modern life. Sophisticated numerical analysis software is commonly embedded in popular software packages. Attaining this level of user transparency requires reliable, efficient, and accurate numerical analysis software, and it requires problem-solving environments PSE in which it is relatively easy to model a given situation. PSEs are usually based on excellent theoretical mathematical models, made available to the user through a convenient graphical user interface. Applications

Computer-aided engineering CAE is an important subject within engineering, and some quite sophisticated PSEs have been developed for this field. A wide variety of numerical analysis techniques is involved in solving such mathematical models. The models follow the basic Newtonian laws of mechanics, but there is a variety of possible specific models, and research continues on their design. One important CAE topic is that of modeling the dynamics of moving mechanical systems, a technique that involves both ordinary differential equations and algebraic equations generally nonlinear. The numerical analysis of these mixed systems, called differential-algebraic systems, is quite difficult but necessary in order to model moving mechanical systems. Building simulators for cars, planes, and other vehicles requires solving differential-algebraic systems in real time. Another important application is atmospheric modeling. In order to create a useful model, many variables must be introduced. Fundamental among these are the velocity  $V(x, y, z, t)$ , pressure  $P(x, y, z, t)$ , and temperature  $T(x, y, z, t)$ , all given at position  $x, y, z$  and time  $t$ . In addition, various chemicals exist in the atmosphere, including ozone, certain chemical pollutants, carbon dioxide, and other gases and particulates, and their interactions have to be considered. The underlying equations for studying  $V(x, y, z, t)$ ,  $P(x, y, z, t)$ , and  $T(x, y, z, t)$  are partial differential equations; and the interactions of the various chemicals are described using some quite difficult ordinary differential equations. Many types of numerical analysis procedures are used in atmospheric modeling, including computational fluid mechanics and the numerical solution of differential equations. Researchers strive to include ever finer detail in atmospheric models, primarily by incorporating data over smaller and smaller local regions in the atmosphere and

implementing their models on highly parallel supercomputers. Modern businesses rely on optimization methods to decide how to allocate resources most efficiently. For example, optimization methods are used for inventory control, scheduling, determining the best location for manufacturing and storage facilities, and investment strategies. Computer software Software to implement common numerical analysis procedures must be reliable, accurate, and efficient. Moreover, it must be written so as to be easily portable between different computer systems. Since about 1950, a number of government-sponsored research efforts have produced specialized, high-quality numerical analysis software. The most popular programming language for implementing numerical analysis methods is Fortran, a language developed in the 1950s that continues to be updated to meet changing needs. Another approach for basic problems involves creating higher level PSEs, which often contain quite sophisticated numerical analysis, programming, and graphical tools. Two popular computer programs for handling algebraic-analytic mathematics manipulating and displaying formulas are Maple and Mathematica. Historical background Numerical algorithms are at least as old as the Egyptian Rhind papyrus c. 1650. Ancient Greek mathematicians made many further advancements in numerical methods. In particular, Eudoxus of Cnidus c. 400 BC. When used as a method to find approximations, it is in much the spirit of modern numerical integration; and it was an important precursor to the development of calculus by Isaac Newton and Gottfried Leibniz. Calculus, in particular, led to accurate mathematical models for physical reality, first in the physical sciences and eventually in the other sciences, engineering, medicine, and business. These mathematical models are usually too complicated to be solved explicitly, and the effort to obtain approximate, but highly useful, solutions gave a major impetus to numerical analysis. Another important aspect of the development of numerical methods was the creation of logarithms about 1614 by the Scottish mathematician John Napier and others. Logarithms replaced tedious multiplication and division often involving many digits of accuracy with simple addition and subtraction after converting the original values to their corresponding logarithms through special tables. Mechanization of this process spurred the English inventor Charles Babbage to build the first computer. Newton created a number of numerical methods for solving a variety of problems, and his name is still attached to many generalizations of his original ideas. Following Newton, many of the mathematical giants of the 18th and 19th centuries made major contributions to numerical analysis. One of the most important and influential of the early mathematical models in science was that given by Newton to describe the effect of gravity. The force on  $m$  is directed toward the centre of gravity of the Earth. Following the development by Newton of his basic laws of physics, many mathematicians and physicists applied these laws to obtain mathematical models for solid and fluid mechanics. Civil and mechanical engineers still base their models on this work, and numerical analysis is one of their basic tools. In the 19th century, phenomena involving heat, electricity, and magnetism were successfully modeled; and in the 20th century, relativistic mechanics, quantum mechanics, and other theoretical constructs were created to extend and improve the applicability of earlier ideas. One of the most widespread numerical analysis techniques for working with such models involves approximating a complex, continuous surface, structure, or process by a finite number of simple elements. Known as the finite element method FEM, this technique was developed by the American engineer Harold Martin and others to help the Boeing Company analyze stress forces on new jet wing designs in the 1940s. FEM is widely used in stress analysis, heat transfer, fluid flow, and torsion analysis. Theory of numerical analysis The following is a rough categorization of the mathematical theory underlying numerical analysis, keeping in mind that there is often a great deal of overlap between the listed areas. Numerical linear and nonlinear algebra Many problems in applied mathematics involve solving systems of linear equations, with the linear system occurring naturally in some cases and as a part of the solution process in other cases. Solving linear systems with up to 1,000 variables is now considered relatively straightforward in most cases. For larger linear systems, there is a variety of approaches depending on the structure of the coefficient matrix  $A$ . Direct methods lead to a theoretically exact solution  $x$  in a finite number of steps, with Gaussian elimination the best-known example. In practice, there are errors in the computed value of  $x$  due to rounding errors in the computation, arising from the finite length of numbers in standard computer arithmetic. Iterative methods are approximate methods that create a sequence of approximating solutions of increasing accuracy. Nonlinear problems are often treated numerically by

reducing them to a sequence of linear problems. This generalizes to handling systems of nonlinear equations. There are numerous other approaches to solving nonlinear systems, most based on using some type of approximation involving linear functions. An important related class of problems occurs under the heading of optimization. Given a real-valued function  $f(x)$  with  $x$  a vector of unknowns, a value of  $x$  that minimizes  $f(x)$  is sought. In some cases  $x$  is allowed to vary freely, and in other cases there are constraints on  $x$ . Such problems occur frequently in business applications.

**Approximation theory** This category includes the approximation of functions with simpler or more tractable functions and methods based on using such approximations. When evaluating a function  $f(x)$  with  $x$  a real or complex number, it must be kept in mind that a computer or calculator can only do a finite number of operations. By including the comparison operations, it is possible to evaluate different polynomials or rational functions on different sets of real numbers  $x$ . The evaluation of all other functions is possible. All function evaluations on calculators and computers are accomplished in this manner. One common method of approximation is known as interpolation. The polynomial  $p(x)$  is said to interpolate the given data points. Interpolation can be performed with functions other than polynomials although these are most common, with important cases being rational functions, trigonometric polynomials, and spline functions made by connecting several polynomial functions at their endpoints. They are commonly used in statistics and computer graphics. Interpolation has a number of applications. If  $n$  is at all large, spline functions are generally preferable to simple polynomials. Most numerical methods for the approximation of integrals and derivatives of a given function  $f(x)$  are based on interpolation. For example, begin by constructing an interpolating function  $p(x)$ , often a polynomial, that approximates  $f(x)$ , and then integrate or differentiate  $p(x)$  to approximate the corresponding integral or derivative of  $f(x)$ .

**Solving differential and integral equations** Most mathematical models used in the natural sciences and engineering are based on ordinary differential equations, partial differential equations, and integral equations. Numerical methods for solving these equations are primarily of two types. The first type approximates the unknown function in the equation by a simpler function, often a polynomial or piecewise polynomial spline function, chosen to closely follow the original equation. The finite element method discussed above is the best known approach of this type. The second type of numerical method approximates the equation of interest, usually by approximating the derivatives or integrals in the equation. The approximating equation has a solution at a discrete set of points, and this solution approximates that of the original equation. Such numerical procedures are often called finite difference methods. Most initial value problems for ordinary differential equations and partial differential equations are solved in this way. Numerical methods for solving differential and integral equations often involve both approximation theory and the solution of quite large linear and nonlinear systems of equations.

**Effects of computer hardware** Almost all numerical computation is carried out on digital computers. The structure and properties of digital computers affect the structure of numerical algorithms, especially when solving large linear systems. First and foremost, the computer arithmetic must be understood.

## 4: Applied Numerical Mathematics - Journal - Elsevier

*Numerical mathematics is the branch of mathematics that proposes, develops, analyzes and applies methods from scientific computing to several fields including analysis, linear algebra, geometry, approximation theory.*

## 5: CRAN Task View: Numerical Mathematics

*Numerical Mathematics: Theory, Methods and Applications (NM-TMA) publishes high-quality original research papers on the construction, analysis and application of numerical methods for solving scientific and engineering problems.*

## 6: Numerical analysis | mathematics | www.enganchecubano.com

*The Journal of Numerical Mathematics (formerly East-West Journal of Numerical Mathematics) contains high-quality papers featuring contemporary research in all areas of Numerical Mathematics. This includes the development, analysis,*

and implementation of new and innovative methods in Numerical Linear Algebra, Numerical Analysis, Optimal Control.

## 7: Numerical Mathematics and Stochastics

BIT was started by Carl Erik Fr  berg in The name is an acronym for "Tidskrift f  r Informationsbehandling" read backwards. From the outset, a wide area of computer science and technology was covered, but since the focus has been on Numerical Mathematics. BIT publishes original research.

## 8: Numerical Mathematics and Scientific Computation - Oxford University Press

The system of linear algebraic equations  $Ax = b$  may or may not have a solution, and if it has a solution, it may or may not be unique. Gaussian elimination is the standard method for solving the linear.

## 9: Numerical Mathematics - Alfio Quarteroni, Riccardo Sacco, Fausto Saleri - Google Books

Mathematical Preliminaries Use the Taylor series for the natural logarithm  $\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$  With  $x = 1$   $\ln 2 = \frac{1}{2} - \frac{1}{4} + \frac{1}{6} - \frac{1}{8} + \dots$  Add the eight terms shown.

*Heywood Massey court of protection practice. Wobblies, pile butts, and other heroes Communication in Nursing (Communication in Nursing (Balzer-Riley)) Mineral water business plan in hindi Butterfly charted designs The birth of financial DNA (r) Top 10 of Everything 2002 Multiplication of distributions Games and Activities for Sign Language Classes One man, one mule, one shovel Good to great jim collins book Aloka prosound alpha 7 manual American critical essays on the Divine comedy. Holy bible tagalog version Nhra 2015 rule book The God of This World to His Prophet What the nurse thought of it Joseph Needham and the science of China, by D. J. de Solla Price. Crazy about her Spanish boss Quick escapes Dallas/Fort Worth Simulation of boreal ecosystem carbon and water budgets A Feminist Philosophy of Religion A conversation with an angel and other essays [microform] Babies and young children in care Tuberculosis in developing countries Music in early America The burden of knowledge and the death of the renaissance man Ocular differential diagnosis Irregular past tense worksheets Japanese-German relations, 1895-1945: war, diplomacy and public opinion Elements of 3-D seismology The federalist papers publius V. 3-4. The history of Pendennis. The Black Baroness The Character Factor Handbook of aboriginal American antiquities El Greco revisited: Candia, Venice, Toledo. The aims of argument Modernity and postmodernity Battle at Ice Palace (Sonic X)*