TWO-POINT BOUNDARY VALUE PROBLEMS: SHOOTING METHODS pdf

1: Two point boundary problem - Shooting method | Physics Forums

It treats the two-point boundary value problem as an initial value problem (IVP), in which xplays the role of the time variable, with abeing the \initial time" and bbeing the \ nal time".

In the case of Robin type, both functional value and derivative of the solutions are given in 2. If only functional value is given, then condition in 2 is known as Dirichlet type; otherwise BVPs will be subject to Neumann condition when only derivative values exist. Robin boundary conditions arise in several branches of applications such as in electromagnetic problem and heat transfer problem where these Robin type conditions are called impedance boundary conditions and the convective boundary conditions are, respectively, as explained in [1]. The Bernoulli polynomial together with Galerkin approximation in solving linear and nonlinear Robin boundary condition problems had been studied by Islam and Shirin [2]. The eminent scholars including Duan et al. Meanwhile, Bhatta and Sastri [5] and Lang and Xu [6] had transformed respective BVPs into discretization form and solved them using symmetric global continuous spline and Quintic B-spline collocation method, respectively. To optimize the computational cost, the development of proposed algorithm must at least satisfy the facility to generate solutions at several points simultaneously as suggested in Fatunla [7]. This implementation has been shown in the literature discussed by Phang et al. They obtained the approximate solution of 1 at two points concurrently. Therefore, the advantages of outcome from their discussion have motivated us in this study. Diagonal block method for solving differential equations was widely studied in the previous literatures. These include the discussion on solving first-order differential equations in [11 , 12]. Solving second-order ordinary differential equations using diagonal block method has been discussed in [13]. The formulation in [12 , 13] is based on backward differentiation approach. In [11], the author has derived the diagonal block method using Lagrange interpolation polynomial for solving first-order ordinary differential equations. In this research, we have extended the derivation in [11] to obtain the formulation of direct integration for solving second-order differential equations. Then the new formulae obtained have been implemented to solve the boundary value problems. The investigation will be focusing on the Newton divided difference interpolation as an iterative technique in seeking as well as updating the missing initial guesses while performing the shooting technique. The organization of this paper is as follows. Section 2 presents the derivation of the two-point diagonal block method. Section 3 elaborates on the analysis of the method including the order, consistency, and stability. Section 4 presents the idea of the procedure of shooting method together with Newton divided difference interpolation as an iterative part. For validation and a clear overview, five tested problems will be discussed in Section 5. Finally, Section 6 concludes the finding from this study. Derivation of the Diagonal Block Method The interval of is divided into a series of blocks so that each block contains two subintervals with equal distance step size, , as depicted in Figure 1. Both numerical solutions of and in these subintervals are simultaneously generated using the appropriate number of back values. The approximate solution of at the point will be computed using three back values at the points, , and.

2: Two Point Boundary Value Problems

If it is a two-point boundary value problem, then there are two classes of numerical methods to solve the problem. The "shooting method" resembles the problem of hitting a target by adjusting the aim of an artillery piece.

See the section on initial value problems for an example of how this is achieved. See also Ascher et al. Such rewritten systems may not be unique and do not necessarily provide the most efficient approach for computational solution. As shown in Ascher et al. Like rewriting the original BVP in the compact form 1 , rewriting a multipoint problem as a two-point problem may not lead to a problem with the most efficient computational solution. Indeed, there is no general theory. However, there is a vast literature on individual cases; see Bernfeld and Lakshmikantham for a survey of a variety of techniques that may be used. Hence, generally there will be none, one or an infinite number of solutions, analogously to the situation with systems of linear algebraic equations. In addition to the possibilities for linear problems, nonlinear problems can also have a finite number of solutions. Consider the following simple model of the motion of a projectile with air resistance: This model neglects threeâ€"dimensional effects such as cross winds and the rotation of the projectile. To see that there is not, consider the case when the target is very close to the cannon. We can hit the target by shooting with an almost flat trajectory or by shooting high and dropping the projectile mortar-like on the target. Shooting or marching methods The approach to proving existence exemplified by the projectile model suggests a computational method of solution. Also, the nonlinear equation may be solved by any suitable method. Since there are quality codes for both tasks this suggests an approach that can be useful in practice. Physical intuition suggests exploiting the relationship between the angle chosen and the range achieved in a bisection-like algorithm but, in more complex cases, such simple physical relationships are usually not available and a general purpose method such as a Newton iteration is often used. The shooting method can be very successful on simple problems such as the projectile problem. It can be extended easily to suggest a method of solution for almost any boundary value problem based on solving equation 3 and it has been automated in many pieces of mathematical software. However, its success depends on a number of factors the most important of which is the stability of the initial value problem that must be solved at each iteration. Unfortunately, it is the case that for many stable boundary value problems the corresponding initial value problems beginning from either endpoint and integrating towards the other endpoint are insufficiently stable for shooting to succeed. So, shooting methods are not computationally suitable for the whole range of practical boundary value problems, particularly those on very long or infinite intervals. Infinite intervals Many ODE BVPs arise from the analysis of partial differential equations through the computation of similarity solutions or via perturbation methods. These problems are often defined on semi-infinite ranges. Of course, the boundary condition at infinity is asymptotic. This problem is easy to solve computationally â€" shooting from the origin and using a standard nonlinear equation solver works without difficulty. In the Blasius problem, the location and type of boundary conditions are determined physically and give us a stable well-conditioned problem. In general, matters are more complicated though physical principles remain an essential guide. This problem provides an example of the requirements of exponential dichotomy; Ascher et al. For a problem to be well-posed the boundary conditions must be set appropriately. For the simple equation 6 , if the boundary conditions are separated, essentially we must have two boundary conditions at the origin and one at infinity matching the two decaying and one increasing towards infinity basis functions in the solution. Numerical methods We described shooting methods above and we explained there that there are inherent problems in this approach. These problems may be overcome, at least partially, using variants on the shooting method which broadly come under the heading of multiple shooting; see Ascher and Petzold Most general purpose software packages for BVPs are based on global methods which fall into two related categories. The resulting difference equations plus the boundary conditions give a set of algebraic equations for the solution on the mesh. These equations are generally nonlinear but are linear when the differential equations and boundary conditions are both linear. To achieve a user-specified error the software generally adjusts the mesh placement using local error estimates based on higher order differencing involving techniques such as deferred correction; see Ascher and Petzold and Shampine et al. A second global approach is to approximate the solution defined in terms of a basis for a linear space of functions usually defined piecewise on a mesh and to collocate this approximate solution. In collocation we substitute the approximate solution in the system of ODEs then require the ODE system to be satisfied exactly at each collocation point. The number of collocation points plus the number of boundary conditions must equal the number of unknown coefficients in the approximate solution; that is, they must equal the dimension of the linear space. The most common choice of approximation is a linear space of splines. For a given linear space, the collocation points must be placed judiciously to achieve optimal accuracy. The error is again controlled by adjusting the mesh spacing using local error estimates involving approximate solutions of varying orders of accuracy; see Ascher et al. Choosing a spline basis for collocation or more or less equivalently using certain types of Runge-Kutta formulas on the mesh leads to a nonlinear system which must be solved iteratively. At each iteration we must solve a structured linear system of equations. When the boundary conditions are separated, the system is almost block diagonal. Similarly structured systems arise from finite difference approximations and also from multiple shooting techniques. Because of the great practical importance of this type of linear algebra problem, significant effort has been devoted to developing stable algorithms which minimize storage and maximize efficiency; see Amodio et al. The case of nonseparated boundary conditions leads to a similarly structured system whose solution poses potentially greater stability difficulties. Sturmâ€"Liouville eigenproblems Another type of BVP that arises in the analytical solution of certain linear partial differential equations is the Sturmâ€"Liouville eigenproblem. ODE eigenvalue problems can be solved using a general-purpose code shooting code that treats an eigenvalue as an unknown parameter. However, with such a code one can only hope to compute an eigenvalue close to a guess. Specialized codes are much more efficient and allow you to be sure of computing a specific eigenvalue; see Pryce for a survey. Numerical methods for Sturmâ€"Liouville eigenproblems that have been implemented in software include finite difference and finite element discretizations which each lead to generalized algebraic eigenproblems where approximations to a number of the lower eigenvalues are available simultaneously. Paprzycki, Almost block diagonal linear systems: Internal references Nestor A. Schmajuk Classical conditioning. Scholarpedia , 3 3: James Meiss Dynamical systems. Shampine and Skip Thompson Initial value problems. Atkinson Numerical analysis. John Butcher Runge-Kutta methods. Philip Holmes and Eric T.

3: Diagonal Block Method for Solving Two-Point Boundary Value Problems with Robin Boundary Condition

Chapter 9 Two point boundary value problems Abstract When differential equations are required to satisfy boundary conditions at more than one value of the independent variable, the resulting problem is called a boundary value.

Ali To cite this article: Pure and Applied Mathematics Journal. In the present paper, a shooting method for the numerical solution of nonlinear two-point boundary value problems is analyzed. Dirichlet, Neumann, and Sturm- Liouville boundary conditions are considered and numerical results are obtained. Numerical examples to illustrate the method are presented to verify the effectiveness of the proposed derivations. The solutions are obtained by the proposed method have been compared with the analytical solution available in the literature and the numerical simulation is guarantee the desired accuracy. Finally the results have been shown in graphically. Method of Solution Technique 4. Results and Discussion 5. Introduction There are many linear and nonlinear problems in science and engineering, namely second order differential equations with various types of boundary conditions, are solved either analytically or numerically. Two-point boundary value problems occur in a wide variety of problem such as modeling of chemical reactions, the boundary layer theory in fluid mechanics and heat power transmission. The wide applicability of boundary value problems in engineering and sciences calls for faster and accurate numerical methods. Many authors have attempted to obtain higher accuracy rapidly by using a numerous methods. The shooting method to compute eigen-values of fourth-order two-point boundary value problems studied by D. Jones [1]. Wang et al [2] investigated application of the shooting method to second order multi point integral boundary value problems. Abd-Elhameed et al [4] have investigated a new wavelet collection method for solving second-order multipoint boundary value problems using Chebyshev polynomials of the third and fourth kinds. See [5] studied nonlinear two point boundary value problem using two step direct method. Meade et al [6] discussed about the shooting technique for the solution two- point boundary value problems. Fatullayev et al [8] investigated numerical solution of a boundary value problem for a second order Fuzzy differential equation. Granas et al [9] investigated the shooting method for the numerical solution of a class of nonlinear boundary value problems. Cole and Adeboye [10] studied an alternative approach to solutions of nonlinear two point boundary value problems. Qiao and Li [11] analyzed two kinds of important numerical methods forcalculating periodic solutions. TrungHieu [12] studied remarks on the shooting method for nonlinear two-point boundary value problem. Russell and Shampine [13] discussed numerical methods for singular boundary value problem. Sharma et al [14] studied numerical solution of two point boundary value problems using Galerkin-Finite element method. Hence the main objective of the present study is to solve nonlinear two point boundary value problems BVP by using simple and efficient shooting method. This well-known technique is an iterative algorithm which attempts to identify appropriate initial conditions for a related initial value problem that provides the solution to the original boundary value problem. Mathematical Formulation For a general boundary value problem for a second-order ordinary differential equation, the simple shooting method is stated as follows:

4: Shooting method - Wikipedia

Shooting-Projection Method for Two-Point Boundary Value Problems Stefan M. Filipov 1, Ivan D. Gospodinov 1, IstvÃ¡n FaragÃ³ 2 1 Department of Computer Science, Faculty of Physical, Mathematical, and Technical Sciences.

Due to the nature of the mathematics on this site it is best views in landscape mode. If your device is not in landscape mode many of the equations will run off the side of your device should be able to scroll to see them and some of the menu items will be cut off due to the narrow screen width. Boundary Value Problems Before we start off this section we need to make it very clear that we are only going to scratch the surface of the topic of boundary value problems. There is enough material in the topic of boundary value problems that we could devote a whole class to it. The intent of this section is to give a brief and we mean very brief look at the idea of boundary value problems and to give enough information to allow us to do some basic partial differential equations in the next chapter. Now, with that out of the way, the first thing that we need to do is to define just what we mean by a boundary value problem BVP for short. With initial value problems we had a differential equation and we specified the value of the solution and an appropriate number of derivatives at the same point collectively called initial conditions. For second order differential equations, which will be looking at pretty much exclusively here, any of the following can, and will, be used for boundary conditions. We will also be restricting ourselves down to linear differential equations. We will, on occasion, look at some different boundary conditions but the differential equation will always be on that can be written in this form. None of that will change. The changes and perhaps the problems arise when we move from initial conditions to boundary conditions. One of the first changes is a definition that we saw all the time in the earlier chapters. If any of these are not zero we will call the BVP nonhomogeneous. It is important to now remember that when we say homogeneous or nonhomogeneous we are saying something not only about the differential equation itself but also about the boundary conditions as well. When solving linear initial value problems a unique solution will be guaranteed under very mild conditions. In that section we saw that all we needed to guarantee a unique solution was some basic continuity conditions. With boundary value problems we will often have no solution or infinitely many solutions even for very nice differential equations that would yield a unique solution if we had initial conditions instead of boundary conditions. In fact, a large part of the solution process there will be in dealing with the solution to the BVP. Or maybe they will represent the location of ends of a vibrating string. So, the boundary conditions there will really be conditions on the boundary of some process. We know how to solve the differential equation and we know how to find the constants by applying the conditions. Example 1 Solve the following BVP. This next set of examples will also show just how small of a change to the BVP it takes to move into these other possibilities. Example 2 Solve the following BVP. Example 3 Solve the following BVP. This, however, is not possible and so in this case have no solution. So, with Examples 2 and 3 we can see that only a small change to the boundary conditions, in relation to each other and to Example 1, can completely change the nature of the solution. All three of these examples used the same differential equation and yet a different set of initial conditions yielded, no solutions, one solution, or infinitely many solutions. Note that this kind of behavior is not always unpredictable however. Also, note that with each of these we could tweak the boundary conditions a little to get any of the possible solution behaviors to show up i. Example 4 Solve the following BVP. All of the examples worked to this point have been nonhomogeneous because at least one of the boundary conditions have been non-zero. Example 6 Solve the following BVP. Example 7 Solve the following BVP. Example 8 Solve the following BVP. Because of this we usually call this solution the trivial solution. Sometimes, as in the case of the last example the trivial solution is the only solution however we generally prefer solutions to be non-trivial. This will be a major idea in the next section. Before we leave this section an important point needs to be made. So, there are probably several natural questions that can arise at this point. The answers to these questions are fairly simple. First, this differential equation is most definitely not the only one used in boundary value problems. It does however exhibit all of the behavior that we wanted to talk about here and has the added bonus of being very easy to solve. So, by using this differential equation almost exclusively we can see and discuss the important behavior that we need to discuss and frees us up from lots of potentially messy solution details and or messy solutions. We will, on occasion, look at other differential equations in the rest of this chapter, but we will still be working almost exclusively with this one. There is another important reason for looking at this differential equation. Admittedly they will have some simplifications in them, but they do come close to realistic problem in some cases.

TWO-POINT BOUNDARY VALUE PROBLEMS: SHOOTING METHODS pdf

5: Boundary value problem - Scholarpedia

1 Introduction The object of my dissertation is to present the numerical solution of two-point boundary value problems. In some cases, we do not know the initial conditions for derivatives of a certain.

A set of n constraints or boundary conditions must be given in order to completely determinethe solutions. Generally these take the form of values of the function y x and its derivatives at the initial and final points of an interval over which solutions are required. But if n1 conditions are given at the initial point x1, y1 x1 , For example, suppose that then two boundary conditions are required for a solution. An n-th order problem can always be reduced to a system of n first-order problems. These methods employ small but finite steps dx, dy1, and dy2 in place of differentials dx, dy1, and dy2. If it is a two-point boundary value problem, then there are two classes of numerical methods to solve the problem. The "shooting method" resembles the problem of hitting a target by adjusting the aim of an artillery piece. We imagine doing this by guessing perhaps randomly values for the functions yi at the initial point those "free" n2 values that are unknown from initial conditions. We then integrate the initial value problem to arrive at the other boundary the target. In general the solution values of yi at the final point will deviate from the required known n2 boundary values there. Our "score" in this shooting practice is determined by the differences between target boundary values and those obtained by integrating. Now we adjust the free initial point parameters to reduce the deviations at the final point. Iterations of this procedure are continued until the desired accuracy is achieved. Relaxation methods use a different approach. These methods are preferred over shooting method when the solutions are smooth and not highly oscillatory. The authors of " Numerical Recipes " self confessed notorious computer gunslingers advise us to "always shoot first, and only then relax". Eigenvalue Problems As is the case for the example introduced above, the boundary value problem may contain a parameter, denoted there by l. Then solutions may not be possible for arbitrary values of l. A slight modification of the procedure is then made using the following trick. The solution is again found by interation starting with a guess for free initial conditions. Denote these n2 values by the components of a vector v. Now, before iteration begins we must guess these component values; the other n1 initial conditions are known.

6: Differential Equations - Boundary Value Problems

In applying Newton's method it is often necessary to choose the initial approximation carefully to ensure convergence, and example is one in ON SHOOTING METHODS FOR BOUNDARY VALUE PROBLEMS which multiple shooting has proved less exacting than ordinary shooting in this respect.

7: Shooting Method for Boundary Value Problems

Two Point Boundary Value Problems boundary value problems. In the shooting method (x) we choose values for all of the dependent variables at one boundary.

TWO-POINT BOUNDARY VALUE PROBLEMS: SHOOTING METHODS pdf

Witness for the dead Synopsis: A Partly Conjectural Outline of Human Evolution Starstruck (Mediterranean Nights) Elegantly Easy Liqueur Desserts Living in the shadow world 52 Ways to Protect Your Teen The articles of association Putting together an investigative team Food microbiology an introduction 4th edition Mistress of the east Language to go elementary book Pro SQL Server 2005 Service Broker (Experts Voice) Teaching children to study Guide to sites and museums. Astrology and Aptitude Pay no attention to that man behind the curtain (the legal industry and how law partnerships work). When the body speaks The Enigma of Ferment Serge lang undergraduate analysis 2nd edition U00a7 105. The Calvinistic Baptists 844 Dreadful Sorry (Point Signature) Fuzzy mathematical programming and fuzzy matrix games Pharmacy list tulsa ok Stupid Men Jokes and Other Laughing Matters Dail and Hammars pulmonary pathology Technology and communication in the humanities Principles of biochemistry 5th edition moran Manual de ciencia politica francisco miro quesada Behavior and handling of ships Daddylonglegs (Bug Books) Curing the tobacco epidemic: the role of workplace tobacco control policies and programs Omowunmi Osinubi The stations of the Cross for the new millennium Gabriels Honor (Secrets!) Through their own eyes Redemption of matter Appendix C: How other churches are using the bod4God program Proceedings of the XXXII International Congress for Asian and North African Studies, Hamburg, 25th-30th A Heart to hart 9 to 5 sheet music To Tame The Hunter (Men: Made In America (Men Made in America Ser.) The Herbal Desk Reference